Answers to Sample Questions

No calculator was used.

1. Suppose that the predicate $P(x)$ means "x is positive". Let the domain of interpretation D be the set $\mathbb{R}$ of all real numbers. Express each of the following sentences in the notation of predicate logic.

   (a) Some real numbers are positive.

   $\exists x\, P(x)$

   (b) All real numbers are positive.

   $\forall x\, P(x)$

2. Suppose that the predicate $P(x)$ means "x is even".

   (a) Let the domain of interpretation D be the set $\mathbb{N}$ of all positive integers. Which of the following are true? Justify your answers.

   (i) $\exists x\, P(x)$

   True. The statement "there exists x such that $P(x)$ is true" means that there exists a positive integer x such that x is even. This is true; for example, we can take $x = 2$.

   (ii) $\forall x\, P(x)$

   False. The statement "for all x, $P(x)$ is true" means that for every positive integer x, x is even. This is false; for example, $x = 1$ is a positive integer which is not even.

   (b) Find a finite domain of interpretation D_1 for which the first statement is true but the second is false.

   (E.g.) $D_1 = \{1, 2, 3, 4\}$
   There exists an even integer in this set, but not every integer in the set is even.

   (c) Find a finite domain D_2 for which both statements are true.

   (E.g.) $D_1 = \{2, 4, 6\}$
   Every element of this set is an even integer.

   (d) Find a finite domain D_3 for which both statements are false.

   (E.g.) $D_1 = \{1, 3, 5\}$
   There are no even integers in this set, so the first statement is false. Also, $x = 1$ is an example of an element in the set which is not an even integer. So the second statement is also false.

   (e) Can you find a set D_4 for which the first statement is false but the second is true? Either describe your set, or explain why no such set exists.

   The null set $\varnothing$ is the only example. There exists no element in the set, so there exists no even integer in the set. This means that the first statement is false. But the second statement is vacuously true, because it's impossible to find a counterexample (namely, an element of the set which is not an even integer).

3. For each of the predicate formulae shown below, apply the generalised de Morgan laws so as to write down a logically equivalent predicate formula in which the "negation" connective appears immediately before the predicate.

   (a) $\sim\forall x\, P(x) \equiv \exists x \sim P(x)$

   (b) $\sim\exists x\, \forall y\, P(x, y) \equiv \forall x\, \exists y \sim P(x, y)$

   (c) $\sim\forall x\, \exists y\, \exists z\, P(x, y, z) \equiv \exists x\, \forall y\, \forall z \sim P(x, y, z)$