

## Answers to Sample Questions

No calculator was used.

1. (a)  $q = \lfloor a/b \rfloor, r = a - qb = a - \lfloor a/b \rfloor b$

(b)  $73 = 6 \cdot 11 + 7 \quad \therefore q = 6 \text{ and } r = 7$

(c)  $60 = 6 \cdot 9 + 6 \quad \therefore q = 6 \text{ and } r = 6$

2. (a)

$$\begin{aligned} 73 &= 6 \cdot 11 + 7 \\ 11 &= 1 \cdot 7 + 4 \\ 7 &= 1 \cdot 4 + 3 \\ 4 &= 1 \cdot 3 + 1 \\ 3 &= 3 \cdot 1 \end{aligned}$$

Since last nonzero remainder is 1,  $\gcd(73, 11) = 1$ .

(b)

$$\begin{aligned} 1 &= 4 - 1 \cdot 3 \\ &= 4 - 1 \cdot (7 - 1 \cdot 4) \\ &= 2 \cdot 4 - 1 \cdot 7 \\ &= 2 \cdot (11 - 1 \cdot 7) - 1 \cdot 7 \\ &= 2 \cdot 11 - 3 \cdot 7 \\ &= 2 \cdot 11 - 3 \cdot (73 - 6 \cdot 11) \\ &= 20 \cdot 11 - 3 \cdot 73 \\ &= -3 \cdot 73 + 20 \cdot 11 \end{aligned}$$

So  $x = -3$  and  $y = 20$ .

3. (a)

$$\begin{aligned} 120 &= 3 \cdot 35 + 15 \\ 35 &= 2 \cdot 15 + 5 \\ 15 &= 3 \cdot 5 \end{aligned}$$

Since last nonzero remainder is 5,  $\gcd(120, 35) = 5$ .

(b)

$$\begin{aligned} 5 &= 35 - 2 \cdot 15 \\ &= 35 - 2 \cdot (120 - 3 \cdot 35) \\ &= 7 \cdot 35 - 2 \cdot 120 \\ &= -2 \cdot 120 + 7 \cdot 35 \end{aligned}$$

So  $x = -2$  and  $y = 7$ .

4. (a)

$k$	$i$	$j$	$j \mid i$	$l$	$i = l$	$p_k$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
9	21	3	T	3	F	
	23	3	F			
		5	F			
		7	F			
		$\vdots$	$\vdots$			
		19	F			
		21	F			
		23	T	23	T	23

(b)	$k$	$i$	$j$	$j i$	$l$	$i=l$	$p_k$
	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	10	25	3	F			
			5	T	5	F	
		27	3	T	3	F	
		29	3	F			
			5	F			
			7	F			
			⋮	⋮			
			23	F			
			27	F			
			29	T	29	T	29