Answers to Sample Questions

No calculator was used.

1. Consider the matrix $A = \begin{pmatrix} 3 & -5 & 7 \\ 0 & 1 & -4 \end{pmatrix}$.

   (a) There are 2 rows in A.

   (b) There are 3 columns in A.

   (c) The order of A is (2×3).

   (d) $-A = \begin{pmatrix} -3 & 5 & -7 \\ 0 & -1 & 4 \end{pmatrix}$

   (e) $A^T = \begin{pmatrix} 3 & 0 \\ -5 & 1 \\ 7 & -4 \end{pmatrix}$

2. Consider the matrix $B = \begin{pmatrix} 3 & -5 \\ 0 & 1 \end{pmatrix}$.

   (a) The order of B is (2×2).

   (b) $B^T = \begin{pmatrix} 3 & 0 \\ -5 & 1 \end{pmatrix}$

   (c) $|B| = det\, B = 3 \cdot 1 - 0 \cdot (-5) = 3$

   (d) No, B isn't singular, because its determinant is nonzero.

   (e) Yes, B is invertible because it's nonsingular. Its inverse is $B^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 5 \\ 0 & 3 \end{pmatrix}$.

3. If C is an invertible (3×3)-matrix, then $CC^{-1} = I = I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

4. Consider the matrices $A = \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -3 & 4 \\ 1 & 0 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 3 & -5 & 4 \end{pmatrix}$ and $D = \begin{pmatrix} 2 & -3 \\ -1 & 0 \end{pmatrix}$.

   (a) (i) $A + A = \begin{pmatrix} 4 \\ 0 \\ -6 \end{pmatrix}$

   (ii) $A - A = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = O$

   (iii) The difference $B - D$ doesn't exist, because B and D have different orders.

   (iv) The product AA doesn't exist, because A is not a square matrix.

   (v) The product AB doesn't exist, because A has 1 column but B has 2 rows.

   (vi) The product AC exists because A has 1 column and C has 1 row:

$$AC = \begin{pmatrix} 6 & -10 & 8 \\ 0 & 0 & 0 \\ -9 & 15 & -12 \end{pmatrix}.$$

   (vii) The product AD doesn't exist, because A has 1 column but D has 2 rows.

(b) $BA = \begin{pmatrix} -6 \\ -4 \end{pmatrix}$

$CA = \begin{pmatrix} -6 \end{pmatrix}$

$DB = \begin{pmatrix} 3 & -6 & 2 \\ -3 & 3 & -4 \end{pmatrix}$

$D^2 = DD = \begin{pmatrix} 7 & -6 \\ -2 & 3 \end{pmatrix}$