Answers to Sample Questions

No calculator was used.

1. (a) $|-3.57| = 3.57$
   (b) Yes, when $x = 0$ or x is negative, $|x| = -x$.

2. (a) $\lfloor -3.8 \rfloor = -4$
   (b) $\lceil -3.8 \rceil = -3$
   (c) $\lfloor 3.8 \rfloor = 3$
   (d) $\lceil 3.8 \rceil = 4$
   (e) Yes, $\lfloor x \rfloor = \lceil x \rceil$ whenever x is an integer, in which case both functions have value x.
   (f) $\lfloor x \rfloor = \begin{cases} \lceil x \rceil \text{ if } x \text{ is an integer} \\ \lceil x \rceil - 1 \text{ if } x \text{ is not an integer} \end{cases}$

3. (a) $9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362880$
   (b) $10! = 10 \cdot 9! = 3628800$
   (c) $\dfrac{2000!}{1999!} = \dfrac{2000 \cdot 1999 \cdot 1998 \cdot \ \ldots \ \cdot 3 \cdot 2 \cdot 1}{1999 \cdot 1998 \cdot \ \ldots \ \cdot 3 \cdot 2 \cdot 1} = 2000$

4. (a) $a^x \cdot a^y = a^{x+y}$
   (b) No, $(2^3)^4 = 8^4 = 2^{12}$ whereas $2^{(3^4)} = 2^{81}$.
   (c) Consider the function $y = 2^x$.
      (i) When $x = 0$, $y = 2^0 = 1$.
      (ii) No, $y = 2^x > 0$ for every real number x.
      (iii) No, for the same reason as above.

5. (a) $\log_2 128 = 7$ since $2^7 = 128$
   (b) $\log_2 1/4 = -2$ since $2^{-2} = 1/4$
   (c) $\log_2 1 = 0$ since $2^0 = 1$
   (d) $\log_2 2^{3596} = 3596$
   (e) If $y = 3^x$, $x = \log_3 y$.
   (f) If $y = \log_5 x$, $x = 5^y$.

6. (a) $deg[f(x)] = 3$
   (b) $f(1) = 1^3 - 5 \cdot 1 + 1 = -3$
   (c) $f(2) = 2^3 - 5 \cdot 2 + 1 = -1$
   (d) No; for example, $f(3) = 3^3 - 5 \cdot 3 + 1 = 13$.
   (e) $deg[g(x)] = 3$
   (f) $f(x) + g(x) = x^2 + 2x + 4$
   (g) $deg[f(x) + g(x)] = 2$
   (h) No, as the above example shows.
   (i) $deg[h(x)] = 1$
   (j) $f(x)h(x) = (x^3 - 5x + 1)(4x - 1) = (x^3 - 5x + 1)(4x) - (x^3 - 5x + 1)(1) = 4x^4 - 20x^2 + 4x - x^3 + 5x - 1 = 4x^4 - x^3 - 20x^2 + 9x - 1$
   (k) $deg[f(x)h(x)] = 4$
   (l) No, as the above example shows.
   (m) $deg[f(x)h(x)] = deg[f(x)] + deg[h(x)]$

7. (a) $\sum_{i=1}^{5} 2^i = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 = 62$
   (b) $\sum_{i=1}^{5} 2^i = \sum_{i=0}^{4} 2^{i+1}$
   (c) $1 + 2 + 3 + 4 = \sum_{i=1}^{4} i$
   (d) $\prod_{i=0}^{3} (2i + 1) = (2 \cdot 0 + 1) \cdot (2 \cdot 1 + 1) \cdot (2 \cdot 2 + 1) \cdot (2 \cdot 3 + 1) = 1 \cdot 3 \cdot 5 \cdot 7 = 105$
   (e) $2 \cdot 4 \cdot 6 \cdot 8 = \prod_{i=1}^{4} (2i)$