Lecture 5

In this lecture we introduce the idea of a function from one set to another.

Functions

Let X and Y be sets.

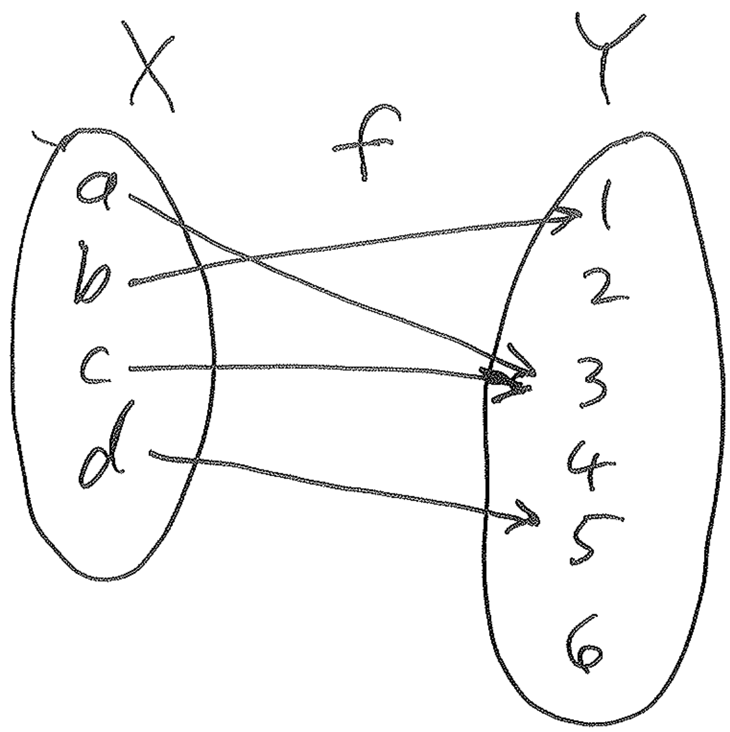
A **function** from X to Y is a rule or procedure that associates an element of Y to every element of X.

We write

$$f : X \longrightarrow Y$$

to mean that f is a function from X to Y.

E.g.



Notice that every element of X has exactly one arrow leaving it.

The set X is called the <u>domain</u> of the function f.

The set Y is called the <u>codomain</u> of f.

Each arrow takes an element of X to its <u>image</u> in Y. This image is denoted by $f(x)$, called "eff of ecks".

So, in this example,

$$f(a) = 3 \ ,$$

$$f(b) = 1 \ ,$$

$$f(c) = 3$$

and

$$f(d) = 5 \ .$$

The subset of Y consisting of all the images of the elements in X is called the range of f.

It's denoted by

$$f(X).$$

So

$$f(X) = \{f(x) : x \in X\}.$$

In our example,

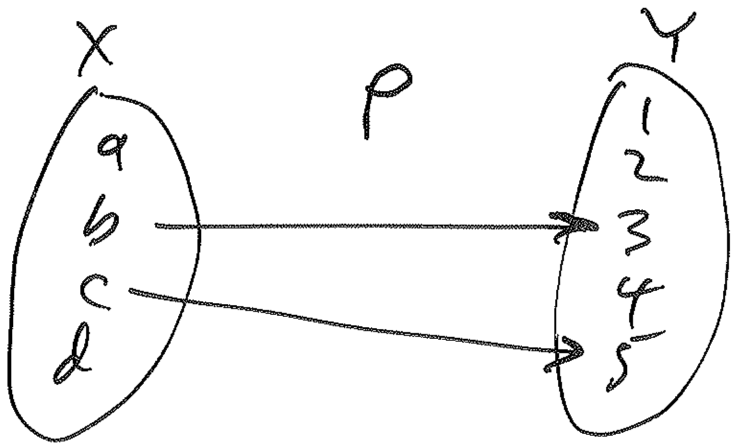
$$\begin{aligned} f(X) &= \{f(a), f(b), f(c), f(d)\} \\ &= \{3, 1, 3, 5\} \\ &= \{1, 3, 5\}. \end{aligned}$$

Note

Some texts use "range" to mean "codomain", and "image" instead of "range".

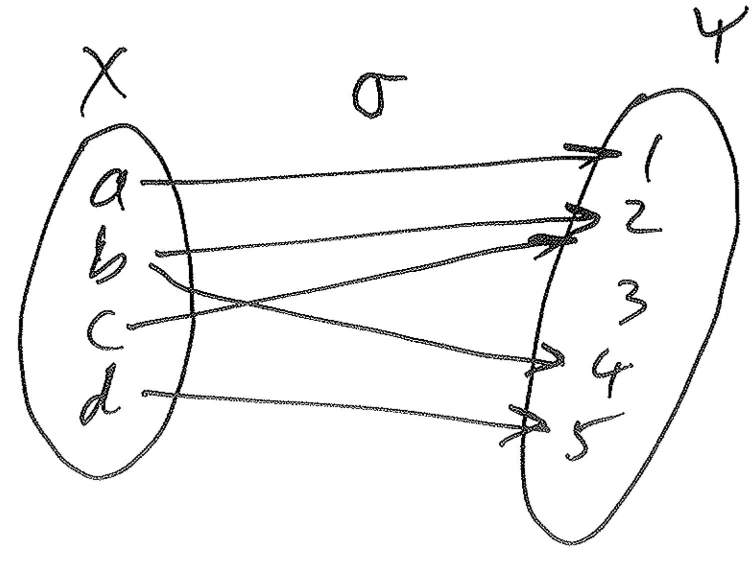
Relations that are not functions

Eg.



Here ρ is not a function because some elements from X have no image.

Eg.



Here, σ (sigma) is not a function because some elements have more than one image.

Note that functions are special kinds of relations.

A function can be regarded as a relation where every element of the first set occurs exactly once as the first coordinate of an ordered pair.

The set

$$\{(x, f(x)) : x \in X\}$$

completely characterises the function f (provided that we know the codomain Y).

For that reason, sometimes we identify the two concepts and write

$$f = \{(x, f(x)) : x \in X\}.$$

In our example,

$$f = \{(a,3),(b,1),(c,3),(d,5)\}.$$

Special kinds of functions

Functions may be:

- one-to-one
- onto
- both one-to-one & onto
- neither one-to-one nor onto