Lecture 3

We continue our study of set theory, and introduce the idea of a relation on a set.

We denote the disjoint union of X & Y by

$$X \dot{\cup} Y.$$

Then

$$X \cup Y = (X \setminus Y) \cup (X \cap Y) \cup (Y \setminus X)$$

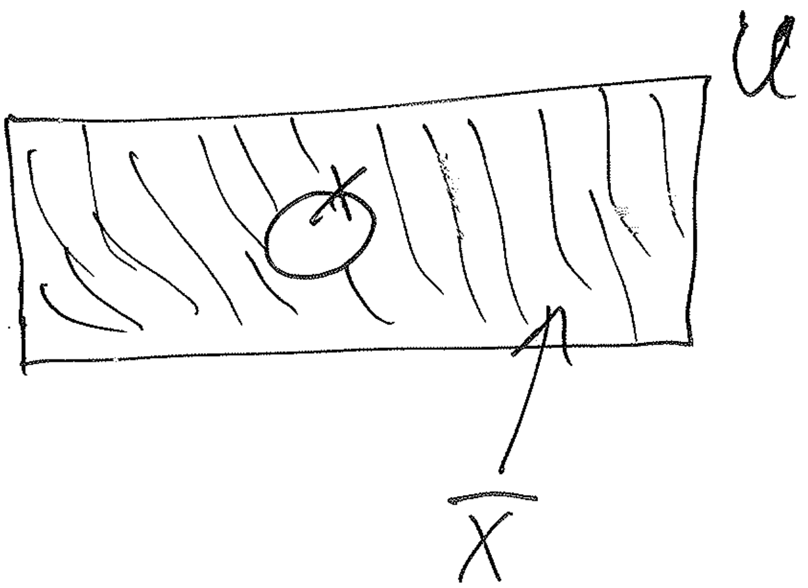
$$= (X - Y) \cup (X \cap Y) \cup (Y - X)$$

$$= [(X - Y) \dot{\cup} (Y - X)] \dot{\cup} (X \cap Y).$$

Often we have a <u>universal</u> set $\mathcal{U}$ consisting of all elements of interest. So every other set of interest is a subset of $\mathcal{U}$.

If $X \subseteq \mathcal{U}$ we write

$$\begin{aligned} \overline{X} &= \mathcal{U} \setminus X \\ &= \mathcal{U} - X \\ &= \text{the } \underline{\text{complement}} \text{ of } X \end{aligned}$$



<u>Lemma</u> (de Morgan's law for sets)

(i) $\overline{X \cup Y} = \overline{X} \cap \overline{Y}$

(ii) $\overline{X \cap Y} = \overline{X} \cup \overline{Y}$

<u>De Morgan's law for Boolean algebra</u>

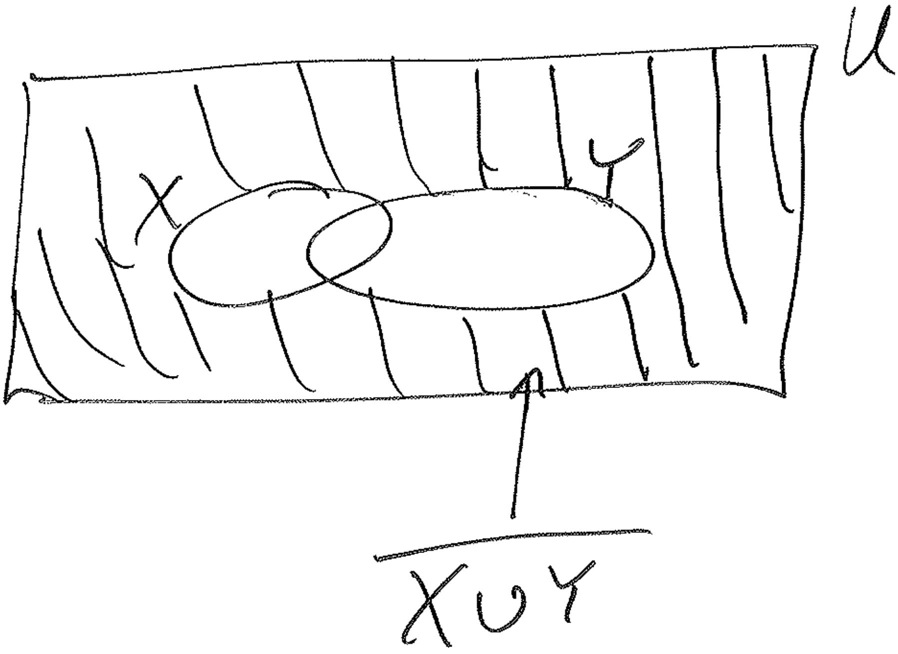
(i) $\overline{x+y} = \overline{x}\,\overline{y}$

(ii) $\overline{xy} = \overline{x} + \overline{y}$

<u>De Morgan's law for logic</u>

(i) not (P or Q) $\equiv$ (not P) and (not Q)

(ii) not (P and Q) $\equiv$ (not P) or (not Q)



Eg. $X = \{1, 2, 3, 4, 5\}$

$Y = \{2, 4, 6, 8\}$

$U = \{1, 2, \ldots, 10\}$

Find:

(a) $X \cup Y$

(b) $X \cap Y$

(c) $\overline{X}$

(d) $\overline{Y}$

(e) $X - Y$

(f) $Y - X$

Sol$^{\underline{n}}$ You do it!

Other sets

$$X = \{x \in \mathbb{Z} \mid x \geq 4\}$$

such that

$$= \{x \in \mathbb{Z} : x \geqslant 4\}$$

$$= \{\text{all integers} \geqslant 4\}$$

$$= \{4, 5, 6, 7, \ldots\}$$

$$Y = \{x \in \mathbb{R} \mid x^2 = 9\}$$

$$= \{-3, 3\}$$

Note For finite sets X and Y,

$$|X \cup Y| = |X| + |Y| - |X \cap Y|.$$

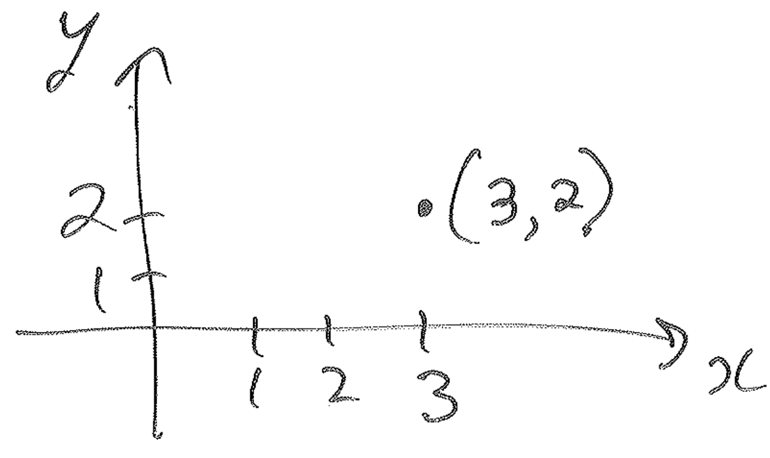
Relations on Sets

Let X, Y be sets.

A <u>relation</u> <u>between</u> X <u>and</u> Y is a subset of the <u>Cartesian product</u>

$$X \times Y = \{(x,y) \mid x \in X, y \in Y\}.$$

So a relation is a set of ordered pairs of the form (x,y), where $x \in X$ and $y \in Y$.



E.g. $X = \{a, b, c\}$, $Y = \{1, 2\}$

$$X \times Y = \{(x, y) \mid x \in X, y \in Y\}$$
$$= \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$$

Any subset of $X \times Y$ is a relation between X and Y.

(E.g.) $R_1 = \emptyset$

$R_2 = \{(a,1), (b,1), (c,1)\}$

$R_3 = \{(c,2)\}$

$\vdots$

(There are 61 more.)

When $X = Y$, a relation between X and Y is called <u>a relation on</u> X.

$$X \times X = X^2 \text{ (usual notation)}$$
$$= \{(x,y) \mid x, y \in X\}$$

Any subset of X^2 is a relation on X.

<u>E.g.</u> $X = \{1, 2, 3, 4\}$

$R_1 = \{(1,1), (2,2), (3,3), (4,4)\}$

$R_2 = \{(1,3), (2,4)\}$

etc.