Lecture 2

We continue our study of elementary set theory.

Notation

$x \in X$

element

$|X|$ = cardinality (size) of X

$\boxed{X \subseteq Y}$ X is contained in Y

(or) X is a subset of Y

This means every element of X is also in Y.

Note Some books use $\subset$ instead of $\subseteq$.

$\emptyset \subseteq \mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$

Note that the null set is regarded as a subset of every set, including itself.

set of all "complex" nos.

not very important for us

We know $\emptyset = \{\ \}$.

What is this? $\underbrace{\{\emptyset\}}$

<u>not</u> the null set

This is a one-element set (a singleton) whose only element is the null set.

This means we can have <u>a set of sets</u> — a set whose elements are sets.

E.g. Let $A = \{a, b, c\} = \{b, c, a\}$.

List all subsets of A.

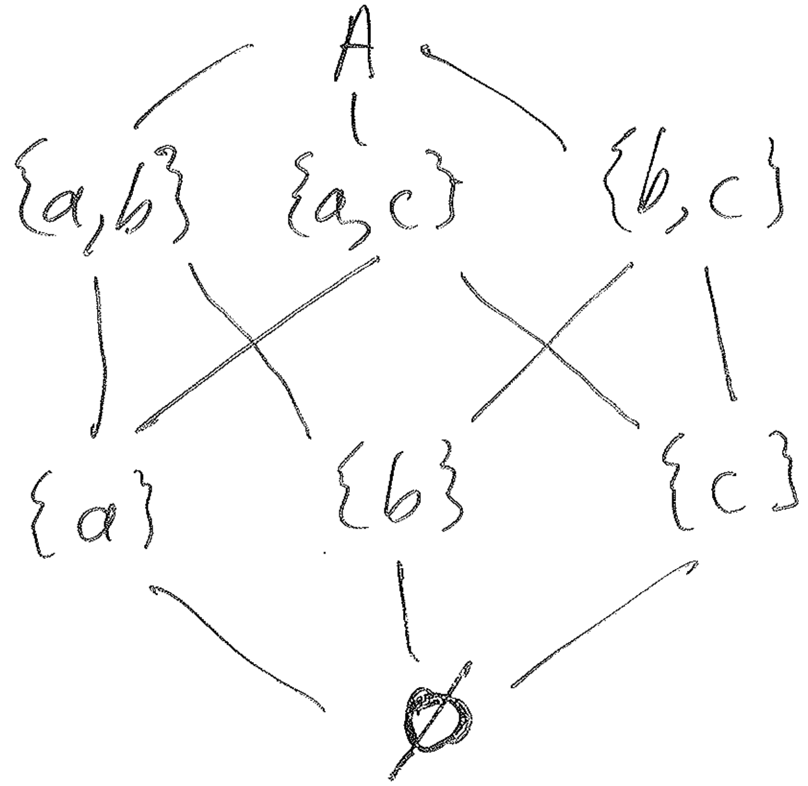
Sol$^{\underline{n}}$

$\emptyset$ — the no-element subset

$\{a\}, \{b\}, \{c\}$ — " 1- " subsets

$\{a,b\}, \{a,c\}, \{b,c\}$ — the 2-element "

$A = \{a,b,c\}$ — the 3-element subset



The 8-element Boolean algebra

Power Sets

The set of all subsets of a given set X is called the power set of X, denoted by $\mathcal{P}(X)$.

E.g., A as above

$$\mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, A\}$$

Note: $|\mathcal{P}(A)| = 8$

Theorem

$$|\mathcal{P}(X)| = 2^{|X|}$$

Proof Omitted.

Operations on Sets

$X \cup Y$ = "ecks union why"

= the <u>union</u> of X and Y

= the set of all elements in X or Y

$X \cap Y$ = "ecks intersection why"

= the <u>intersection</u> of X and Y

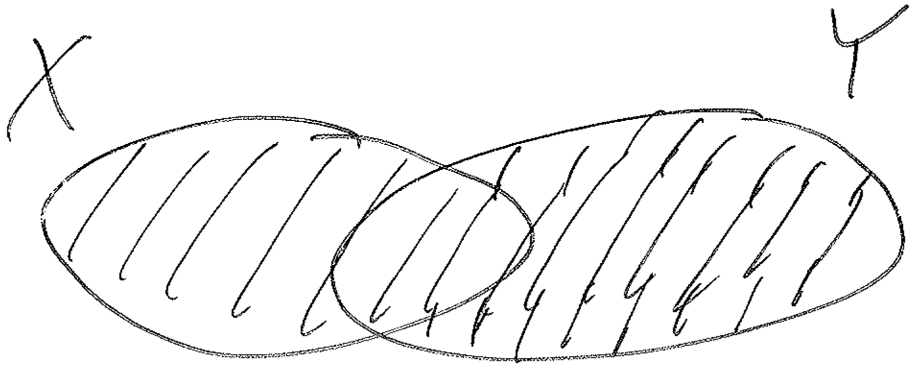
= the set of all elements in X and Y

<u>E.g.</u>

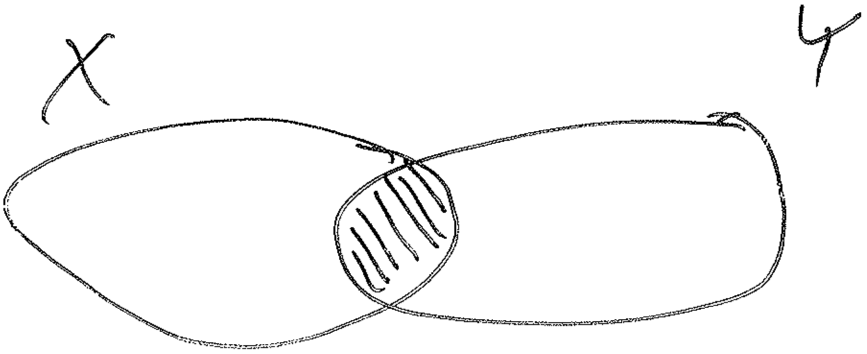
$$X = \{a, b, c, d, e\}$$
$$Y = \{c, d, e, f\}$$

$$X \cup Y = \{a, b, c, d, e, f\}$$
$$X \cap Y = \{c, d, e\}$$

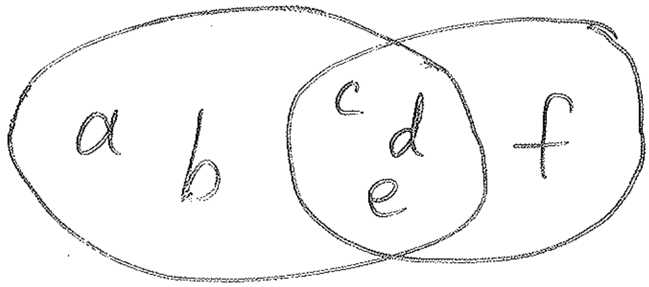
Venn Diagrams



$X \cup Y$ is shaded



$X \cap Y$ is shaded



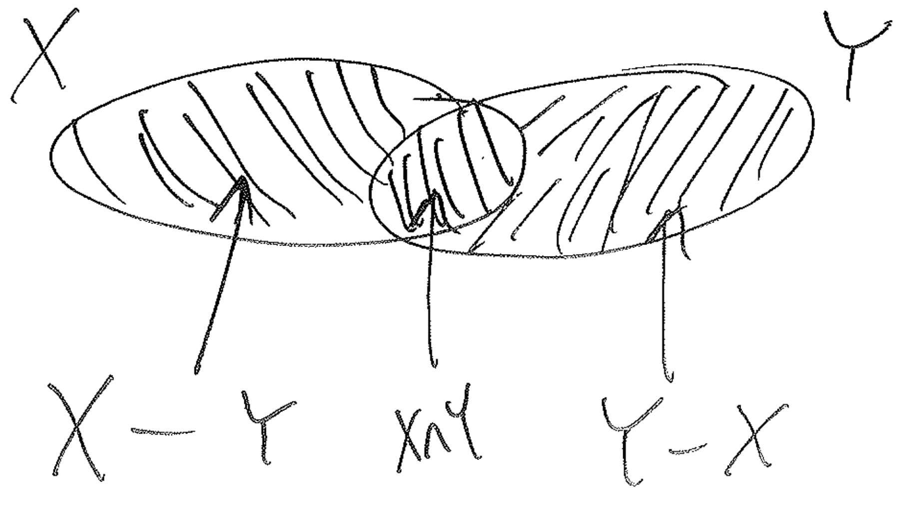
$X - Y$ = the complement of Y relative to X

= the "set difference"

$= X \setminus Y$

(alternative notation)

$= \{x \in X : x \notin Y\}$



Two sets are disjoint if they don't intersect.

If $X \cap Y = \emptyset$ then $X \cup Y$ is called the disjoint union of X and Y.