TAYLOR'S COLLEGE PETALING JAYA
SCHOOL OF COMPUTING

RMIT BACHELOR OF COMPUTER SCIENCE
RMIT BACHELOR OF INFORMATION TECHNOLOGY

JUNE 2010
SUPPLEMENTARY EXAMINATION

Subject (Code)	**: Mathematics for Computing (MATH 2111)**
Day & Date	**: ??**
Time (Duration)	**: ?? (2 hours)**
Reading Time	**: ??**
No. of Pages	**: 5 (including cover page)**

Instructions to Candidates:

1. Place your student ID with photograph on the desk for inspection by the invigilator.
2. Complete the front cover of the examination answer booklet. Write the question numbers attempted on the front cover of the examination answer booklet.
3. Detach and place the completed attendance slip on the desk for collection by the invigilator.
4. Ensure that you have received the correct and complete set of questions for the examination.
5. This paper is preceded by a 15-minute reading time. Do not start writing your answers during this time.
6. Answer **ALL EIGHT (8)** questions in the answer booklet. 200 marks
7. Programmable calculators are not permitted during this examination. However, non-programmable calculators are permitted during this examination.
8. To obtain full marks for a question, full working should be shown.
9. This is a closed book examination.
10. At the end of the examination, **do not remove the question paper** or any examination stationery, used or unused, from the examination hall.
11. Severe disciplinary action will be taken against those caught violating examination rules.
12. This examination contributes 60% of your final mark in MATH 2111.

Answer **ALL** questions.

Question 1

Let A = set of all integers. For a positive integer n, we define $B_n = \{\text{all positive multiples of } n\} = \{kn \mid k \text{ is a positive integer}\}$.

(a) Describe the elements in the following sets concisely:

(i) $A - B_1$

(ii) $B_2 \cap B_4$.

(iii) $(A - B_2) \cup (A - B_4)$.

(b) Give an example of an element in the Cartesian product, $A \times A \times A$.

(c) Find the number of bijective functions from the set $\{0, 1, 2, 3\}$ to $\{a, b, c, d\}$.

(d) Find the number of 1-to-1 functions from the set $\{0, 1, 2, 3, 4\}$ to $\{a, b, c\}$.

(e) Consider the relation R defined on A by the following: For all $a, b \in A$,

$$a \, R \, b \Leftrightarrow a - b \text{ is an even number.}$$

(i) Show that R is a reflexive relation.

(ii) Show that R is a transitive relation.

(iii) Suppose that R is an equivalence relation. Describe the equivalence classes of R.

((5 x 3) + 5 + 5 + 5 + (5 x 3) = 45 marks)

Question 2

Given that $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

(a) Give the order of the transpose of A.

(b) Compute the product BA.

(5 + 5 = 10 marks)

Question 3

(a) Consider the statement:

Bala does not stay in Kuala Lumpur or he walks to campus.

(i) Convert the statement into an *if-then* statement.

(ii) Write the contrapositive of that statement.

(b) Let p, q be statement variables. Construct a truth table for $(\sim p \vee q) \Rightarrow \sim q$.

(c) Let p, r, s, t, u, w be statement variables. Assume that the following statements are true:

$$\sim p \Rightarrow r \wedge \sim s$$
$$t \Rightarrow s$$
$$u \Rightarrow \sim p$$
$$\sim w$$
$$u \vee w$$

Use argument forms and suitable rules of inference to deduce whether or not that $\sim t$ is true. Show all your steps clearly.

(d) Let *x* and *y* be any rational numbers. Let $P(x, y)$ be the predicate:

$$P(x, y):\ x^2 - y^2 \leq 0.$$

(i) Write the negation of the statement, $\forall x\ \forall y\ P(x, y)$.

(ii) Determine whether or not the statement, $\forall x\ \exists y\ P(x, y)$ is true. Explain your reasoning.

((5 x 2)+ 8 + 12 +(5 x 2) = 40 marks)

Question 4

(a) Apply the Euclidean algorithm to find integers *a* and *b*, such that 100*a* + 31*b* =1.

(b) Suppose *a* and *b* are integers whose product is divisible by 7. Argue that either the remainder of *a* or *b* is 0 when divided by 7. Your argument should work for general values of *a* and *b*.

(10 + 10 = 20 marks)

Question 5

Consider the sequence $\{a_n\}_{n\geq 0}$ defined as follows:

$$a_n = \sum_{k=0}^{n} \left(\frac{-1}{3}\right)^k$$

(a) Write a recurrence relation for the sequence.

(b) Prove by induction that for all integers $n \geq 1$, $a_n = \frac{3}{4}\left[1 - \left(\frac{-1}{3}\right)^{n+1}\right]$.

(5+ 15 = 20 marks)

Question 6

(a) Let A and B be events with probabilities: $P(A) = 0.60$, $P(B) = 0.40$ and $P(A \cup B) = 0.80$.

(i) Compute $P(A \cap B)$.

(ii) Are A and B independent events? Explain your reasoning.

(b) Suppose we have a finite set of numerical data and each value is added 5. What is the effect on the variance of the original set of data? Explain your reasoning. Your argument should work for a finite but general set of data.

((5+5)+ 10 = 20 marks)

Question 7

(a) Given the random variable, X, with the following probability distribution:

x	1	2	3
$f(x)$	0.1	0.2	0.7

(i) What is the expected value of X?

(ii) What is the variance of X?

(b) Within the city limits of Sydney, police plan to install camera traps to detect speeding at 4 different locations: L_1, L_2, L_3 and L_4. The cameras at each of the locations are operational 50%, 20%, 15% and 20% of the time respectively. Furthermore, the probabilities that a diver exceeds the speed limit at the various locations are 0.20, 0.10, 0.50 and 0.20 respectively.

(i) What is the probability that a driver is actually detected by the camera traps for exceeding the city's speed limit?

(ii) Assuming a driver is caught on camera for exceeding the speed limit, what is the probability that he is caught by the camera trap placed at location, L_2 ?

((5x2)+ (5 x2) = 20 marks)

Question 8

A bit string is a finite sequence of 0's and 1's. For example, 010101011 is a bit string of length 9.

a) How many bit strings of length 9 are there?
b) How many bit strings of length 9 begin with 0 and end with 1?
c) How many bit strings of length 9 have exactly three 1's?
d) How many bit strings of length 5 have two 1's in succession?

((5x 3) + 10 = 25 marks)

END OF PAPER