TAYLOR'S COLLEGE PETALING JAYA
SCHOOL OF COMPUTING

RMIT BACHELOR OF COMPUTER SCIENCE
RMIT BACHELOR OF INFORMATION TECHNOLOGY

JUNE 2010
MAIN EXAMINATION

Subject (Code)	**: Mathematics for Computing (MATH 2111)**
Day & Date	**: ??**
Time (Duration)	**: ?? (2 hours)**
Reading Time	**:**
No. of Pages	**: 5 (including cover page)**

Instructions to Candidates:

1. Place your student ID with photograph on the desk for inspection by the invigilator.
2. Complete the front cover of the examination answer booklet. Write the question numbers attempted on the front cover of the examination answer booklet.
3. Detach and place the completed attendance slip on the desk for collection by the invigilator.
4. Ensure that you have received the correct and complete set of questions for the examination.
5. This paper is preceded by a 15-minute reading time. Do not start writing your answers during this time.
6. Answer **ALL EIGHT (8)** questions in the answer booklet. 200 marks
7. Programmable calculators are not permitted during this examination. However, non-programmable calculators are permitted during this examination.
8. To obtain full marks for a question, full working should be shown.
9. This is a closed book examination.
10. At the end of the examination, **do not remove the question paper** or any examination stationery, used or unused, from the examination hall.
11. Severe disciplinary action will be taken against those caught violating examination rules.
12. This examination contributes 60% of your final mark in MATH 2111.

Answer **ALL** questions.

Question 1

Let $A = \{1, 2, 4, 6, 8, ..., \text{all positive multiples of } 2\}$,

$B = \{1, 3, 6, 9, 12, 15, ..., \text{all positive multiples of } 3\}$ and

$C = \{1, 9, 18, 27, 36\}$.

(a) Describe the elements in the following sets concisely:

(i) $B \setminus A$

(ii) $A \cap B$

(b) How many elements are there in the power set $\wp(C \text{ x } C)$?

(c) Describe a bijective function from A to B.

(d) Is there a 1-to-1 function from B to C? If there are such functions, construct one such example; otherwise explain the obstruction.

(e) Consider the relation R defined on A by the following: For all $a, b \in A$,

$$a \, R \, b \Leftrightarrow a \text{ divides } b.$$

Is R an equivalence relation? Justify your reasoning.

(f) Determine the number of partitions of the following set, $\{0,1,2\}$. Justify your reasoning.

((5 x 2) + 5 + 5 + 5 + 10 + 5 = 40 marks)

Question 2

Given that $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & a \\ 1 & 1 \end{bmatrix}$ and C, a matrix with real entries.

(a) Give two different orders of C where the product, AC exists.

(b) Determine all values of a such that the determinant of B is positive.

(c) Find all the inverse matrices of B.

(4 + 5 + 6 = 15 marks)

Question 3

(a) Write the negation of the following statement by changing the relevant universal quantifiers to existential quantifiers.

For all positive integers a and b, a divides b if and only if b divides a.

(b) Let p and q be statement variables. If $p \vee \sim q$ is false, then determine whether or not the following is true:

$$p \wedge (p \Leftrightarrow \sim q).$$

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(c) Suppose we have a compound proposition which has 245 distinct statement variables. How many different cases in a truth table do we have to examine in order to determine whether or not the compound proposition is a tautology, contradiction or contingency? Justify your answer.

(d) Let x be any rational number. Let $P(x)$ be the predicate:

$$P(x): \lfloor x \rfloor + \lfloor -x \rfloor = 0.$$

Recall that the symbol, $\lfloor x \rfloor$ means the floor of the number, x. Determine whether or not the following statement, $\forall x, P(x)$ is true. Explain your reasoning.

(5 + 10+5 + 5 = 25 marks)

Question 4

Consider the sequence $\{a_n\}_{n\geq 1}$ defined as follows:

$$a_n = 2a_{n-1} + n \text{ for all integers } n \geq 1,$$
$$a_0 = 1.$$

(a) Compute $\sum_{k=0}^{3} a_k$.

(b) Compute $\prod_{k=0}^{3} a_k$.

(c) Is $a_k = 2^k + k$ for all integers $k \geq 0$? Justify your answer.

(d) Use mathematical induction to argue that $a_k \geq 0$ for all integers $k \geq 0$.

(5 + 5 + 5 + 15= 30 marks)

Question 5

(a) A car dealer wishes to buy a used car for which the probabilities are 0.22, 0.36, 0.28 and 0.14, respectively, that he will be able to sell it for a profit of $250, sell it for a profit of $150, break even, or sell it for a loss of $150.

(i) What is expected value of his profit?

(ii) What is the standard deviation of his profit?

(b) Suppose we have a finite set of numerical data and each value is multiplied by 2 and then added 5. What is the effect on the mean of the new set of data as compared to the initial set of data? Explain your reasoning. Your argument should work for a finite but general set of data.

((5+10)+ 10 = 25 marks)

Question 6

(a) A row of guns fires on a target one after another. Each has a probability 1/3 of hitting the target on a given shot.

(i) Compute the probability that the third hit comes before the eighth gun fires.

(ii) Suppose we fire 10 shots in total. Let X be the number of times we hit the target. Compute the mean and variance of X.

(b) Suppose that 40% of all road accidents involve at least one of the drivers driving over the speed limit and that 30% involve at least one of the drivers being drunk. If a drunk driver is involved in an accident, then there is a 60% chance that at least one of the drivers exceeded the speed limit.

(i) What is the probability that an accident involves over speeding and drunk drivers?

(ii) Assuming an accident involving over speeding has occurred, compute the probability that one of the drivers is drunk.

((5+10) + (5 +5) = 25 marks)

Question 7

A security system requires its passwords to have exactly 6 characters of which 5 of them must be alphabets (A, B,…, Z) and 1 of them is a digit (0,1,…,9). Assume that the passwords are not case-sensitive.

(a) How many passwords are possible?

(b) How many passwords consist of three C's, one D's and end in an odd digit?

(c) Suppose we forget our password but remember that it has the characteristics described in (ii), what is the probability that we will guess the password correctly on the first trial?

(d) What is the probability that AAAAA1 is **not** a password?

(5 x 4 marks = 20 marks)

Question 8

(a) Let a and b be positive integers such that $a = 17b + 3$, where $3 < b$. Argue that the greatest common divisor of a and b is either 1 or 3.

(b) Compute the least common multiple of 78936 and 35136.

(10 + 10 = 20 marks)

END OF PAPER –